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## Nucleon Spin and the Mixing of Axial Vector Mesons<sup>1</sup>

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### Abstract

It is shown that the QCD anomaly may lead to an abnormal mixing behavior of the axial vector mesons similar to the pseudoscalar mesons. These mixing effects, involving a gluonic axial vector state, generate a non-vanishing strange quark component in the nucleon. They reduce the matrix element of the singlet axial vector in comparison to the value obtained in a naïve quark model. The results are in agreement with the data obtained in the polarized lepton–nucleon scattering experiments.

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## 1. INTRODUCTION

Deep inelastic scattering reveals that the nucleon is a complex system consisting of an infinite number of quarks, antiquarks and gluons. There is no doubt that the same is true for all mesons and baryons. Nevertheless it seems that under certain circumstances they behave as if they were composed of a single constituent quark and another constituent antiquark or three constituent quarks. Examples are the magnetic moments of the baryons, the spectroscopy of mesons and baryons, the meson–baryon couplings, the ratios of total cross sections like  $\sigma(\pi N) / \sigma(N N)$  etc. . Thus it seems to make sense to decompose the proton into three parts, into three constituent quarks called U or D. A proton would have the composition (UUD). The quantum numbers of the constituent quarks would provide the internal quantum numbers of the nucleon.

In deep inelastic scattering one observes that a nucleon has the composition  $|uud\bar{q}q...g... >$  (g: gluon, q = u,d,s); i. e., the quark density functions (which are scale dependent) are described by a valence quark and an indefinite number of quark–antiquark pairs. One might be tempted to identify the valence quark, defined by the corresponding quark density function, with a constituent quark. This identification would imply that the three–quark picture denoted above is nothing but a very rough approximation and both  $\bar{q}q$ –pairs and gluons need to be added to the picture. In this case, however, one would not be able to understand why the model of a baryon consisting of three constituent quarks works so well in many circumstances. It seems much more likely that a constituent quark is a quasiparticle which has a non-trivial internal structure on its own, i. e., consisting of a valence quark, of many  $\bar{q}q$ –pairs and of gluons. Thus a constituent quark has an effective mass, an internal size, etc. Such an interpretation of a constituent quark is not new [1]. Nevertheless it is still unclear to what extent it can be derived from the basic laws of QCD since it is deeply rooted in the non-perturbative aspects of QCD, in particular the confinement problem. In two dimensions the constituent quarks can be identified with specific soliton solutions of the QCD field equations [2].

One way to gain deeper insights into the internal structure of the constituent quarks is to consider their spin. In the constituent quark picture it is, of course, assumed that the nucleon spin is provided by the combination of the spins of the three constituent quarks. If the latter have a non-trivial internal structure, the question arises whether also the spin structure of the constituent quarks is a complex phenomenon, as it seems to be the case for the nucleon, or not. A simple model for the spin structure would be to assume that the spin of, say, a constituent u-quark is provided by the valence u-quark inside it and the  $\bar{q}q$ -cloud and the gluonic cloud does not contribute to the spin. It will be one of our conclusions that this naïve picture is not correct.

In a naïve  $SU(6)$  quark model of the baryons the spin of the proton is composed of the spins of the three constituent quarks (see, e. g., [3]). The wave function in flavor and spin space is given by

$$|P \uparrow\rangle = \frac{1}{\sqrt{6}} |UUD(2 \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)\rangle. \quad (1.1)$$

Using this wave function it is straightforward to calculate the matrix elements of the spin operators of the various quark flavors in the proton. If we define the quantity

$$\Delta Q \equiv \langle P \uparrow | \sigma_z^Q | P \uparrow \rangle, \quad (1.2)$$

one finds:

$$\Delta U = \frac{4}{3}, \quad \Delta D = -\frac{1}{3}, \quad \Delta S = 0. \quad (1.3)$$

Since the quantity  $\Delta S$  vanishes according to the wave function given above one obtains:

$$\Delta U + \Delta D + \Delta S = 1. \quad (1.4)$$

As expected, the spin of the proton is carried by the spins of the three constituent quarks.

We should like to point out that the same calculation gives for the axial vector coupling

constant, observed in  $\beta$ -decay:

$$g_A/g_V = \Delta U - \Delta D = 5/3, \quad (1.5)$$

while the observed value is 25% smaller:  $g_A/g_V \cong 1.257$  [4]. If we interpret this phenomenon as the result of the depolarisation of a constituent quark by relativistic and by gluonic effects, one expects a reduction factor of 25% for all spin densities. Correspondingly we would expect that the sum of the spin densities given in Eq. (1.4) does not give 1, but rather 0.75. Thus one finds that about 75% of the nucleon spin is carried by the spin of the constituent quarks while 25% are carried by orbital and gluonic effects [5].

These values disagree with the measurements of the spin density functions of the quarks carried out in the recent years [6]. In QCD the first moment of the structure function  $g_1^p$  can be expressed in terms of the sum of the nucleon matrix elements of the axial vector currents, weighted by the square of the quark charges:

$$\int_0^1 dx g_1^p(x, Q^2) = \frac{1}{2} \left[ \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right] + 0(\alpha_S/\pi). \quad (1.6)$$

The experimental data, combined with the experimental knowledge of the axial vector coupling constants for  $\beta$ -decay and hyperon decay, give according to a recent analysis [7]:

$$\begin{aligned} \Delta u &= 0.83 \pm 0.03, & \Delta d &= -0.43 \pm 0.03, \\ \Delta s &= -0.10 \pm 0.03, & \Delta \Sigma &= \Delta u + \Delta d + \Delta s = 0.31 \pm 0.07. \end{aligned} \quad (1.7)$$

These values are obtained at a renormalization scale  $Q^2 = 10$  GeV $^2$ . They disagree with the expectations of the naïve quark model which give in particular  $\Delta S = 0$  and  $\Delta \Sigma \simeq 0.75$ .

The sum  $\Delta \Sigma$  of the three density moments is described by the nucleon matrix element of the singlet axial vector current. Unlike the divergencies of the axial vector currents of the  $SU(3)$  octet the divergence of the singlet axial vector current does not vanish in

the chiral limit, but exhibits an anomaly because of the interaction of quarks with the gluons [8]. It has been suggested that the anomalous behavior of the singlet axial vector current is the reason for the observed smallness of the singlet axial vector nucleon matrix element [9], [10]. Nevertheless it remained unresolved in which way the gluon anomaly influences the spin densities. On the other hand it is well known that the gluonic anomaly of the singlet axial vector current is responsible for the anomalous mixing behavior of the pseudoscalar mesons [11]. It implies, for example, that in the  $SU(3)$  limit the  $\eta$  meson is an  $SU(3)$  octet, while the  $\eta'$  meson is an  $SU(3)$  singlet. The mass difference between these two pseudoscalar mesons is a measure for the impact of the gluonic anomaly on the mass spectrum. In the chiral limit  $SU(3)_L \times SU(3)_R$  the eight pseudoscalar mesons act as Nambu–Goldstone particles and are massless while the  $\eta'$  meson remains massive with a mass of order 1 GeV. The matrix elements of the axial vector currents of the  $SU(3)$  octet exhibit a Goldstone pole and obey a Goldberger–Treiman relation [12]. No such relation exists for the matrix element of the singlet axial vector current. This suggests that the anomalous mixing behavior of the pseudoscalar mesons and the anomalous value for the nucleon matrix element of the singlet axial vector current are related [10] and that also the axial vector mesons might display an anomalous mixing behavior.

Usually it is assumed that the mixing of the axial vector mesons is similar to the mixing of the vector mesons; i. e., the mass eigenstates of the two neutral isoscalar members of the nonet segregate according to the quark decomposition  $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$  and  $\bar{s}s$  [3]. It is well known that this is the case for the vector mesons being a consequence of the Zweig rule. In the case of the axial vector mesons the experimental situation is less clear. In this paper we should like to study the situation of the axial vector mesons in view of the spin problem. In particular we should like to investigate whether the anomalously small matrix element for the singlet axial vector current could be correlated with an anomalous mixing behavior of the axial vector mesons.

## 2. AXIAL VECTOR MESONS AND THEIR MIXING

Before turning to the axial vector mesons, we consider briefly the pseudoscalar mesons. Here the large departure from the ideal mixing is interpreted as the consequence of the existence of strong transitions between the various flavor combinations [11], [13]. This suggests the ansatz for the quadratic mass matrix of the pseudoscalar mesons in the basis  $\{|\bar{u}u\rangle, |\bar{d}d\rangle, |\bar{s}s\rangle\}$ :

$$M_{\bar{q}q}^2 = \begin{pmatrix} m_{\bar{u}u}^2 + \lambda_P & \lambda_P & \lambda_P \\ \lambda_P & m_{\bar{d}d}^2 + \lambda_P & \lambda_P \\ \lambda_P & \lambda_P & m_{\bar{s}s}^2 + \lambda_P \end{pmatrix}. \quad (2.1)$$

The parameter  $\lambda_P$  characterizes the strength of the transitions between the various flavor eigenstates. Here  $m_{\bar{q}q}^2$  describes the mass of the corresponding meson in the absence of the gluonic mixing parameter  $\lambda_P$ . In the  $SU(3)$  limit the masses of the three flavor states are identical. They vanish in the chiral limit. It is useful to consider the basis  $\{\frac{1}{\sqrt{2}}|\bar{u}u - \bar{d}d\rangle, \frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle, |\bar{s}s\rangle\}$ . In the limit of isospin symmetry the state  $\frac{1}{\sqrt{2}}|\bar{u}u - \bar{d}d\rangle$  represents the  $\pi^0$  meson. It does not mix with the other two states because of isospin symmetry and will be disregarded.

Eliminating the  $\pi^0$ -state one finds in the basis  $\{\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle, |\bar{s}s\rangle\}$ :

$$M^2 = \begin{pmatrix} m_N^2 + 2\lambda_P & \sqrt{2}\lambda_P \\ \sqrt{2}\lambda_P & m_S^2 + \lambda_P \end{pmatrix}. \quad (2.2)$$

Here is

$$m_N^2 = m_{\bar{u}u}^2 = m_{\bar{d}d}^2 = m_\pi^2 = (135 \text{ MeV})^2. \quad (2.3)$$

The parameter  $m_S^2$  describes the mass splitting within the octet:

$$m_S^2 = m_{\bar{s}s}^2 = 2m_K^2 - m_\pi^2 = (691 \text{ MeV})^2. \quad (2.4)$$

The mixing angle  $\Phi$  between the state  $|S\rangle = |\bar{s}s\rangle$  and the isosinglet state  $|N\rangle = \frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$  is given by the relation

$$\tan 2\Phi = \frac{2\sqrt{2}\lambda_P}{\lambda_P + m_N^2 - m_S^2}. \quad (2.5)$$

A good description of the experimental situation is obtained for  $\lambda_P = 0.25 \text{ GeV}^2$  [11]. Thus the system of the pseudoscalar mesons exhibits a strong gluonic mixing term which provides the reason for the strong departure from the ideal mixing situation.

In the case of the vector mesons  $\omega$  and  $\Phi$  one observes a very small mixing between the strange system  $|\bar{s}s\rangle$  and the non-strange system  $\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$ . One finds that the gluonic mixing parameter  $\lambda_V$  for the vector mesons is very small compared to  $\lambda_P$ :  $\lambda_P \simeq 18\lambda_V$  [14].

Below we shall study the mixing behavior of the axial vector mesons with the quantum numbers  $J^{PC} = 1^{++}$ . The spectrum of the  $(1^{++})$  mesons consists of the isovector mesons  $a_1(1260)$  and the isoscalar mesons  $f_1(1285)$ ,  $f_1(1420)$  and  $f_1(1510)$  [4]. The strange mesons  $K_1(1270)$  and  $K_1(1400)$ , which constitute an isodoublet, are mixtures of the corresponding  $J^{PC} = (1^{++})-$  and  $J^{PC} = (1^{+-})-$ -eigenstates.

The mass of the  $a_1$  is still subject to a considerable uncertainty [4]. To render our subsequent discussion independent of this we shall use a rounded mass with large error bounds thus covering the whole spectrum of mass candidates:

$$m_{a_1} = (1200 \pm 110) \text{ MeV}. \quad (2.6)$$

Similarly, for the three isoscalar  $f_1$  mesons we shall use the mass values

$$\begin{aligned} m_{f_1^{(1)}} &= (1280 \pm 30) \text{ MeV}, & m_{f_1^{(2)}} &= (1410 \pm 20) \text{ MeV}, \\ m_{f_1^{(3)}} &= (1510 \pm 20) \text{ MeV}. \end{aligned} \quad (2.7)$$

Within a  $SU(3)$  nonet one expects only two isoscalar mesons and we reach the conclusion that one of the three states is not a  $\bar{q}q$  meson, but rather an exotic state (gluonic meson, multiquark state). Due to strong mixing effects one could expect that none of the three isoscalar states is purely of exotic nature, but all three states are mixtures involving  $\bar{q}q$ -parts and exotic parts in their wave function.

The mixing behavior of the strange isodoublet has been under discussion for a long time [15]. In accordance with the predominating opinion, we shall assume a mixing angle between the  $(1^{++})$ – and  $(1^{+-})$ –eigenstates of  $45^\circ$ . This mixing pattern implies that the mass eigenvalue of the strange isodoublet in the  $(1^{++})$ –octet is determined by the average value of the  $K_1(1270)$ – and  $K_1(1400)$ –states (in [mass] $^2$ ):

$$m_{K_1} = (1340 \pm 30) \text{ MeV}. \quad (2.8)$$

In analogy to the pseudoscalar mesons we shall investigate the mixing pattern of the axial vector mesons. In order to accommodate an exotic state, we shall extend the mass matrix given in Eq. (2.1). To be more specific, we shall assume that the exotic configuration in the axial vector channel is of gluonic nature denoted by  $|G\rangle$ . The mass matrix of the axial vector mesons has then the form:

$$M_0^2 = \begin{pmatrix} m_N^2 + \lambda & \lambda & \lambda & \kappa \\ \lambda & m_N^2 + \lambda & \lambda & \kappa \\ \lambda & \lambda & m_S^2 + \lambda & \kappa \\ \kappa & \kappa & \kappa & m_G^2 \end{pmatrix} \quad (2.9)$$

$$m_N^2 = m_{a_1}^2, \quad m_S^2 = 2m_{K_1}^2 - m_{a_1}^2.$$

Again, the parameter  $\lambda$  describes the strength of the mixing between the various  $\bar{q}q$ –configurations while the parameter  $\kappa$  describes the transition between a  $\bar{q}q$ –configuration and the gluonic configuration. Of course, in the special case  $\kappa = 0$  the mass  $m_G$  corresponds to the mass of the physical gluonic state. In Eq. (2.9) we did not denote a contribution of  $\kappa$  in the 44–matrix element since it is included in  $m_G^2$ .

We shall denote the coefficients of the various  $f_1$  mesons in the basis  $\{|N\rangle = \frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle, |S\rangle = |\bar{s}s\rangle, |G\rangle\}$  by  $x_i, y_i, z_i$ :

$$|f_1^{(i)}\rangle = x_i|N\rangle + y_i|S\rangle + z_i|G\rangle. \quad (2.10)$$

In order to calculate these coefficients, we shall proceed in an analogous way as in the case of the pseudoscalar mesons. After excluding the isotriplet configuration, which must remain unmixed because of isospin symmetry, we obtain from Eq. (2.9)

$$M^2 = \begin{pmatrix} m_N^2 + 2\lambda & \sqrt{2}\lambda & \sqrt{2}\kappa \\ \sqrt{2}\lambda & m_S^2 + \lambda & \kappa \\ \sqrt{2}\kappa & \kappa & m_G^2 \end{pmatrix}. \quad (2.11)$$

The mass matrix of the mass eigenvalues is denoted by

$$M_m^2 = \begin{pmatrix} m_{f_1^{(1)}}^2 & 0 & 0 \\ 0 & m_{f_1^{(2)}}^2 & 0 \\ 0 & 0 & m_{f_1^{(3)}}^2 \end{pmatrix}. \quad (2.12)$$

Let us define the quantities  $l_i$ :

$$\begin{aligned} l_1 &\equiv \text{tr } M^2 = \text{tr } M_m^2, \\ l_2 &\equiv \frac{1}{2} [(\text{tr } M^2)^2 - \text{tr}(M^2)^2] \\ &= m_{f_1^{(1)}}^2 m_{f_1^{(2)}}^2 + m_{f_1^{(1)}}^2 m_{f_1^{(3)}}^2 + m_{f_1^{(2)}}^2 m_{f_1^{(3)}}^2, \\ l_3 &\equiv \det M^2 = \det M_m^2. \end{aligned} \quad (2.13)$$

For the parameters  $\lambda$ ,  $\kappa$  and  $m_G$  we then obtain:

$$\lambda = a/b, \quad \kappa = \sqrt{(c\lambda^2 + d\lambda + e)/(m_N^2 + 2m_S^2)}, \quad m_G = \sqrt{l_1 - m_N^2 - m_S^2 - 3\lambda}, \quad (2.14)$$

where

$$\begin{aligned}
a &\equiv l_1(m_N^2 + m_S^2) - m_N^4 - m_S^4 - l_2 - m_N^2 m_S^2 + \\
&\quad + 3(m_S^4 m_N^2 + m_N^4 m_S^2 + l_3 - m_N^2 m_S^2 l_1)/(m_N^2 + 2m_S^2), \\
b &\equiv 2(m_N^2 - m_S^2)^2/(m_N^2 + 2m_S^2), \\
c &\equiv -3(m_N^2 + 2m_S^2), \\
d &\equiv m_N^2(l_1 - m_N^2) + m_S^2(2l_1 - 6m_N^2 - 2m_S^2), \\
e &\equiv m_N^2(l_1 m_S^2 - m_N^2 m_S^2 - m_S^4) - l_3.
\end{aligned} \tag{2.15}$$

The unitary matrix

$$U = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \tag{2.16}$$

describes the transformation between the basis  $\{|N\rangle, |S\rangle, |G\rangle\}$  and the basis  $\{|f_1^{(1)}\rangle, |f_1^{(2)}\rangle, |f_1^{(3)}\rangle\}$ . The equation  $UM^2U^{-1} = M_m^2$  then leads to the mixing parameters

$$\begin{aligned}
x_i &= \sqrt{2}\kappa \left( m_S^2 - m_{f_1^{(i)}}^2 \right) c_i, \\
y_i &= \kappa \left( m_N^2 - m_{f_1^{(i)}}^2 \right) c_i, \\
z_i &= \left[ 2\lambda^2 - \left( m_N^2 + 2\lambda - m_{f_1^{(i)}}^2 \right) \left( m_S^2 + \lambda - m_{f_1^{(i)}}^2 \right) \right] c_i.
\end{aligned} \tag{2.17}$$

The normalization constants  $c_i$  are given by

$$\begin{aligned}
c_i &= \left\{ 2\kappa^2 \left( m_S^2 - m_{f_1^{(i)}}^2 \right)^2 + \kappa^2 \left( m_N^2 - m_{f_1^{(i)}}^2 \right)^2 + \right. \\
&\quad \left. + \left[ 2\lambda^2 - \left( m_N^2 + 2\lambda - m_{f_1^{(i)}}^2 \right) \left( m_S^2 + \lambda - m_{f_1^{(i)}}^2 \right) \right]^2 \right\}^{-\frac{1}{2}}.
\end{aligned} \tag{2.18}$$

Using the meson masses mentioned previously we calculate the numerical results for the coefficients  $x_i$  etc. as shown in Table 1. The uncertainties in the coefficients are relatively large because of the fact that the meson masses are not precisely known.

Table 1: Coefficients of the  $f_1$  mesons.

	$f_1^{(1)}$	$f_1^{(2)}$	$f_1^{(3)}$
$x_i$	$0.93 \pm 0.05$	$0.09 \pm 0.17$	$-0.30 \pm 0.11$
$y_i$	$-0.20 \pm 0.14$	$-0.55 \pm 0.23$	$-0.74 \pm 0.19$
$z_i$	$-0.25 \pm 0.13$	$0.76 \pm 0.19$	$-0.51 \pm 0.23$
$\lambda$ [GeV $^2$ ]	$0.10 \pm 0.04$		
$\kappa$ [GeV $^2$ ]	$0.10 \pm 0.03$		
$m_G$ [MeV]	$1432 \pm 38$		

We find a mixing behavior of the  $f_1$  mesons which is quite different from the mixing behavior of the vector mesons. Such a conclusion has also been reached after an analysis of the radiative decays  $J/\psi \rightarrow \gamma f_1(1285)$ , observed by the Mark III collaboration [16]. Relatively large mixing exists between the  $|N\rangle_-$ ,  $|S\rangle_-$  and  $|G\rangle_-$  states. This corresponds to a relatively large violation of the OZI–rule. Let us, as an illustrative example, consider the coefficients for the masses  $m_{a_1} = 1215$  MeV,  $m_{K_1} = 1320$  MeV,  $m_{f_1^{(1)}} = 1275$  MeV,  $m_{f_1^{(2)}} = 1390$  MeV,  $m_{f_1^{(3)}} = 1540$  MeV as given in Eq. (2.19). The coefficients are in agreement with the observations of the various decays of the  $f_1$  mesons [4]. For instance, the  $f_1^{(2)}$  meson decays dominantly into  $K\bar{K}\pi$  and the meson  $f_1^{(3)}$  into  $K\bar{K}^*(892) + c.c.$ . This is expected since according to Eq. (2.19) both states have a relatively large  $\bar{s}s$ –component.

$$\begin{aligned}
 |f_1^{(1)}\rangle &\simeq 0.89 \left| \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \right\rangle - 0.25 |\bar{s}s\rangle - 0.38 |G\rangle, \\
 |f_1^{(2)}\rangle &\simeq 0.15 \left| \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \right\rangle - 0.63 |\bar{s}s\rangle + 0.76 |G\rangle, \\
 |f_1^{(3)}\rangle &\simeq -0.42 \left| \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \right\rangle - 0.74 |\bar{s}s\rangle - 0.52 |G\rangle.
 \end{aligned} \tag{2.19}$$

According to Eq. (2.19) the meson  $f_1^{(2)}$  has a probability of 58% to be a gluonic meson while this probability is reduced for the  $f_1^{(1)}$  meson to 14 % and for the  $f_1^{(3)}$  meson to 27 %.

The strong mixing in the axial vector meson channel which we obtain reminds us of the corresponding situation for the pseudoscalar mesons. Since in the latter case the  $U(1)$  anomaly is responsible for the large mixing behavior we conclude that the gluonic anomaly also influences the mixing pattern of the axial vector mesons.

Using  $SU(3)$  symmetry, it is also useful to describe the meson mixing in terms of the basis  $\{\frac{1}{\sqrt{6}}|\bar{u}u+\bar{d}d-2\bar{s}s\rangle, \frac{1}{\sqrt{3}}|\bar{u}u+\bar{d}d+\bar{s}s\rangle, |G\rangle\}$ . The coefficients of the  $f_1$  mesons in this basis are

$$\begin{aligned}\tilde{x}_i &= (x_i - \sqrt{2}y_i)/\sqrt{3}, \\ \tilde{y}_i &= (\sqrt{2}x_i + y_i)/\sqrt{3}, \\ \tilde{z}_i &= z_i.\end{aligned}\tag{2.20}$$

### 3. MATRIX ELEMENTS OF THE AXIAL VECTOR CURRENTS

In this section we shall calculate the proton matrix elements of the various axial vector currents using the idea of axial vector dominance. In analogy to the case of vector meson dominance we shall assume that the matrix elements of the axial vector currents are dominated by the contribution of the lowest lying axial vector mesons. Thus we obtain the relation

$$\langle p|\bar{q}\gamma_\mu\gamma_5q|p\rangle = \sum_A \frac{\langle 0|\bar{q}\gamma_\mu\gamma_5q|A\rangle\langle Ap|p\rangle}{m_A^2 - k^2} \Big|_{k=0}.\tag{3.1}$$

Here the matrix element  $\langle 0|\bar{q}\gamma_\mu\gamma_5q|A\rangle$  denotes the transition element of the axial vector current between the vacuum state and the corresponding axial vector meson while the second factor  $\langle Ap|p\rangle$  describes the coupling of the axial vector meson to the proton. The

four-momentum transfer is denoted by  $k$ . The summation in Eq. (3.1) is carried out over all axial vector mesons with the quantum numbers  $J^{PC} = 1^{++}$  which can couple to the proton, i. e., the mesons  $a_1, f_1^{(1)}, f_1^{(2)}$  and  $f_1^{(3)}$ . Once these matrix elements are known we can calculate  $\Delta q$ .

First we consider the matrix element of the third component of the isovector

$$\sqrt{2}\langle 0|A_\mu^3|a_1\rangle = \langle 0|\frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5u - \bar{d}\gamma_\mu\gamma_5d)|\frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)\rangle. \quad (3.2)$$

The decay constant for the  $a_1$  is defined by

$$i\varepsilon_\mu^r f_{a_1} \equiv \sqrt{2}\langle 0|A_\mu^3|a_1\rangle \quad (3.3)$$

( $\varepsilon_\mu^r$ : polarisation vector for the  $a_1$  meson). This equation can be rewritten as follows:

$$i\varepsilon_\mu^r \cdot 2f_{a_1} = \langle 0|\bar{u}\gamma_\mu\gamma_5u|(\bar{u}u)_{a_1}\rangle + \langle 0|\bar{d}\gamma_\mu\gamma_5d|(\bar{d}d)_{a_1}\rangle. \quad (3.4)$$

Here we denote by  $(\bar{u}u)_{a_1}$  the  $\bar{u}u$ -part of the  $a_1$  meson etc. Using  $SU(3)$  symmetry we can define the decay constants of the axial vector meson of quark composition  $(\bar{q}q)$  by

$$i\varepsilon_\mu^r f_A = \langle 0|\bar{q}\gamma_\mu\gamma_5q|(\bar{q}q)_A\rangle. \quad (3.5)$$

The matrix element for the coupling of the meson with the nucleon  $\langle a_1 p|p\rangle$  can be written in terms of the Dirac wave functions:

$$\langle a_1 p|p\rangle = ig_{a_1 pp}\bar{u}(p)\gamma^\nu\gamma_5u(p)\varepsilon_\nu^r, \quad (3.6)$$

where  $g_{a_1 pp}$  denotes the coupling constant of the meson to the nucleon.

If we consider one of the  $f_1$  mesons, we must take into account the mixing of these neutral mesons among each other. In what follows we shall denote the octet state by  $|f_8\rangle = \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle$  and the singlet state by  $|f_0\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle$ . For the corresponding decay constants we shall assume  $f_{f_0} = f_{f_8} = f_{a_1}$  and for the coupling constants  $g_{f_0 pp} = \sqrt{2}g_{f_8 pp}$ .

Taking into account our description for the mixing of the mesons, we obtain

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | f_1^{(i)} \rangle = \tilde{x}_i \langle 0 | \bar{q} \gamma_\mu \gamma_5 q | f_8 \rangle + \tilde{y}_i \langle 0 | \bar{q} \gamma_\mu \gamma_5 q | f_0 \rangle + \tilde{z}_i \langle 0 | \bar{q} \gamma_\mu \gamma_5 q | G \rangle. \quad (3.7)$$

The last term in this equation refers to the transition of the axial vector current to a gluonic state. We shall assume in accordance with the Zweig rule that the transition of the axial vector current, which is a quark bilinear, to the gluonic state vanishes.

Subsequently we discuss the numerical results. We determine the decay constant  $f_{a_1}$  in considering the decay  $\tau^- \rightarrow a_1^- + \nu_\tau$  [17]. We find

$$f_{a_1} = (0.19 \pm 0.03) \text{ GeV}^2. \quad (3.8)$$

The coupling constant  $g_{a_1 pp}$  is difficult to determine directly. However, we can relate it to the observed axial vector coupling constant using axial vector dominance:

$$\frac{1}{6} \frac{g_A}{g_V} = \frac{1}{6} (\Delta u - \Delta d) = \frac{f_{a_1} g_{a_1 pp}}{3\sqrt{2} m_{a_1}^2}. \quad (3.9)$$

Using this constraint, we can determine the  $a_1$ -nucleon coupling constant:

$$g_{a_1 pp} = 6.7 \pm 1.0. \quad (3.10)$$

The coupling constants of the various axial vector mesons to the baryons can be described in terms of reduced elements  $F$  and  $D$ , using  $SU(3)$  symmetry (see, e. g., [18]). One has

$$\begin{aligned} g_{a_1^- \bar{n} p} &= g_{a_1^+ \bar{p} n} = \sqrt{2}(F + D), \\ g_{a_1 pp} &= -g_{a_1 nn} = (F + D), \\ g_{f_8 pp} &= g_{f_8 nn} = \sqrt{3}F - D/\sqrt{3}. \end{aligned} \quad (3.11)$$

Unlike the corresponding  $SU(3)$  matrix elements for the axial vector currents, the reduced matrix elements for the coupling of the axial vector mesons to the baryons are not known. We can estimate them by using the same relation between  $F$  and  $D$  as inferred from the measurements for the corresponding reduced matrix elements for the axial vector currents [4]:

$$\frac{F}{D} = 0.575 \pm 0.016. \quad (3.12)$$

In this case we get

$$g_{f_8 pp} = 1.8 \pm 0.4, \quad g_{f_0 pp} = 2.5 \pm 0.5. \quad (3.13)$$

Subsequently we shall calculate  $\Delta q$ , the singlet sum  $\Delta \Sigma$  and the first moment of the structure function  $g_1^p$  taking into account the mixing behavior of the axial vector mesons<sup>1</sup>. As an illustration we shall first consider a hypothetical situation in which  $SU(3)$  is considered to be exact and the three neutral axial vector mesons are denoted by  $|a_1\rangle$ ,  $|f_8\rangle$  and  $|f_0\rangle$  ( $a_1$  and  $f_8$  are degenerate in the symmetry limit). One finds for the density moments:

$$\begin{aligned} \Delta u^{(1)} &= \frac{1}{\sqrt{6}} \frac{f_{f_8} g_{f_8 pp}}{m_{f_8}^2} + \frac{1}{\sqrt{3}} \frac{f_{f_0} g_{f_0 pp}}{m_{f_0}^2} + \frac{1}{\sqrt{2}} \frac{f_{a_1} g_{a_1 pp}}{m_{a_1}^2}, \\ \Delta d^{(1)} &= \frac{1}{\sqrt{6}} \frac{f_{f_8} g_{f_8 pp}}{m_{f_8}^2} + \frac{1}{\sqrt{3}} \frac{f_{f_0} g_{f_0 pp}}{m_{f_0}^2} - \frac{1}{\sqrt{2}} \frac{f_{a_1} g_{a_1 pp}}{m_{a_1}^2}, \\ \Delta s^{(1)} &= -\frac{2}{\sqrt{6}} \frac{f_{f_8} g_{f_8 pp}}{m_{f_8}^2} + \frac{1}{\sqrt{3}} \frac{f_{f_0} g_{f_0 pp}}{m_{f_0}^2}. \end{aligned} \quad (3.14)$$

It is easy to see that the strange density moment  $\Delta s$  vanishes in the symmetry limit if  $f_0$  and  $f_8$  are degenerate. If the singlet state is heavier than the octet state, a non-zero contribution for  $\Delta s$  is generated which is expected to be negative. In the formal limit  $m_{f_0} \rightarrow \infty$  we obtain

$$\begin{aligned} \Delta u^{(1)} &= 0.77 \pm 0.18, & \Delta d^{(1)} &= -0.56 \pm 0.13, & \Delta s^{(1)} &= -0.20 \pm 0.05, \\ \Delta \Sigma^{(1)} &= 0, & \int_0^1 dx g_1^{p,(1)}(x) &= 0.13 \pm 0.03, \end{aligned} \quad (3.15)$$

results which exhibit the correct qualitative features of the experimental data discussed previously. The singlet sum  $\Delta \Sigma$  vanishes because of the vanishing of the contribution of the singlet axial vector meson  $f_0$  in the limit  $m_{f_0} \rightarrow \infty$ .

It is also useful to consider the following case with the finite mass  $m_{f_0}$ . If we take as

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<sup>1</sup>Due to the anomalous dimension for the singlet axial vector current, the density moments are scale dependent. This scale dependence, however, is relatively weak and therefore does not affect our results within the error bounds (see, e. g., [19]).

an illustrative example  $m_{f_0} = m_{f_1^{(3)}}$ , we get in the case of  $SU(3)$  symmetry

$$\begin{aligned}\Delta u^{(1)} &= 0.89 \pm 0.22, & \Delta d^{(1)} &= -0.43 \pm 0.10, & \Delta s^{(1)} &= -0.07 \pm 0.03, \\ \Delta \Sigma^{(1)} &= 0.39 \pm 0.13, & \int_0^1 dx g_1^{p,(1)}(x) &= 0.17 \pm 0.04.\end{aligned}\quad (3.16)$$

As compared to the previous case, the  $|\Delta s|$  is reduced by more than a factor of 2 while  $\Delta \Sigma$  increases considerably. Of course, comparing these values with the experimental data is not useful at this stage since  $SU(3)$  breaking has not yet been taken into account. We proceed to do so by replacing the states  $|f_0\rangle$  and  $|f_8\rangle$  by the states  $|f_1^{(i)}\rangle$ . In this case the moments of the density functions become

$$\begin{aligned}\Delta u^{(2)} &= \frac{f_{f_8} g_{f_8 pp}}{\sqrt{6}} \sum_{i=1}^3 \frac{1}{m_{f_1^{(i)}}^2} (\tilde{x}_i^2 + 2\tilde{y}_i^2 + 2\sqrt{2}\tilde{x}_i\tilde{y}_i) + \frac{f_{a_1} g_{a_1 pp}}{\sqrt{2}m_{a_1}^2}, \\ \Delta d^{(2)} &= \frac{f_{f_8} g_{f_8 pp}}{\sqrt{6}} \sum_{i=1}^3 \frac{1}{m_{f_1^{(i)}}^2} (\tilde{x}_i^2 + 2\tilde{y}_i^2 + 2\sqrt{2}\tilde{x}_i\tilde{y}_i) - \frac{f_{a_1} g_{a_1 pp}}{\sqrt{2}m_{a_1}^2}, \\ \Delta s^{(2)} &= -\frac{2}{\sqrt{6}} f_{f_8} g_{f_8 pp} \sum_{i=1}^3 \frac{1}{m_{f_1^{(i)}}^2} (\tilde{x}_i^2 - \tilde{y}_i^2 + \tilde{x}_i\tilde{y}_i/\sqrt{2}).\end{aligned}\quad (3.17)$$

As expected we obtain the same contributions as obtained previously in the  $a_1$ -channel while the contributions of the  $f_1$  mesons are modified by the mixing terms. In the special case  $\tilde{x}_i = \delta_{1i}$ ,  $\tilde{y}_i = \delta_{2i}$  we can reconstruct the case discussed above. Using the numerical results from section 2 we find:

$$\begin{aligned}\Delta u^{(2)} &= 0.92 \pm 0.21, & \Delta d^{(2)} &= -0.38 \pm 0.09, & \Delta s^{(2)} &= -0.02 \pm 0.01, \\ \Delta \Sigma^{(2)} &= 0.52 \pm 0.13, & \int_0^1 dx g_1^{p,(2)}(x) &= 0.18 \pm 0.04.\end{aligned}\quad (3.18)$$

It is typical for this case that one obtains a rather small contribution  $|\Delta s|$  and a relatively large value of  $\Delta \Sigma$ .

Thus far we have not taken into account the direct coupling of the gluonic state to the nucleon in assuming  $\langle Gp|p\rangle = 0$ . In view of the fact that gluons contribute a large part of the momentum of a fast moving nucleon such a constraint is highly unrealistic. As soon as

a direct coupling of the nucleon to the gluonic state is introduced by setting  $\langle Gp|p \rangle \neq 0$ , one finds that the density moments have the form

$$\begin{aligned}\Delta u^{(3)} &= \Delta u^{(2)} + \frac{f_{f_8} g_{Gpp}}{\sqrt{6}} \sum_{i=1}^3 \frac{1}{m_{f_1^{(i)}}^2} \tilde{z}_i (\tilde{x}_i + \sqrt{2} \tilde{y}_i), \\ \Delta d^{(3)} &= \Delta d^{(2)} + \frac{f_{f_8} g_{Gpp}}{\sqrt{6}} \sum_{i=1}^3 \frac{1}{m_{f_1^{(i)}}^2} \tilde{z}_i (\tilde{x}_i + \sqrt{2} \tilde{y}_i), \\ \Delta s^{(3)} &= \Delta s^{(2)} + \frac{f_{f_8} g_{Gpp}}{\sqrt{6}} \sum_{i=1}^3 \frac{1}{m_{f_1^{(i)}}^2} \tilde{z}_i (-2\tilde{x}_i + \sqrt{2} \tilde{y}_i).\end{aligned}\quad (3.19)$$

Of course, the coupling constant  $g_{Gpp}$  is not known. We shall treat it as a free parameter. As an example we use  $g_{Gpp} = 19$  :

$$\begin{aligned}\Delta u^{(3)} &= 0.83 \pm 0.20, \quad \Delta d^{(3)} = -0.48 \pm 0.11, \quad \Delta s^{(3)} = -0.10 \pm 0.03, \\ \Delta \Sigma^{(3)} &= 0.25 \pm 0.15, \quad \int_0^1 dx g_1^{p,(3)}(x) = 0.15 \pm 0.04.\end{aligned}\quad (3.20)$$

As one can see, we find a relatively good agreement between observation and the results of axial vector meson dominance, provided the mixing and a relatively large non-vanishing coupling of the gluonic state of the nucleon is taken into account. In particular we find a negative contribution to  $\Delta s$ . The sign of the strange density moment is determined by the same mechanism as in the hypothetical case discussed previously where it arises because of the non-degeneracy of the singlet and the octet states. Since the octet is lower in mass than the singlet, the negative sign of the  $\bar{s}s$ -component in the octet state leads to the negative sign of the strange density moment. Of course, the opposite effect would be expected for the unrealistic case where the singlet state has a smaller mass than the octet state. A good description of the experimental situation is obtained, if the mixing among the neutral axial vector mesons is described according to the mixing scheme discussed above and if the nucleon has a fairly strong coupling to the gluonic state.

It is also instructive to observe that in the hypothetical limit in which no mixing between the various  $\bar{q}q$ -axial vector mesons takes place ( $\lambda = \kappa = 0$ ) one has

$$\tilde{x}_1 = \tilde{y}_2 = \frac{1}{\sqrt{3}}, \quad \tilde{y}_1 = -\tilde{x}_2 = \sqrt{\frac{2}{3}}, \quad (3.21)$$

$$\tilde{x}_3 = \tilde{y}_3 = \tilde{z}_1 = \tilde{z}_2 = 0, \quad \tilde{z}_3 = 1.$$

The masses of the  $f_1$  mesons are given by  $m_{f_1^{(1)}}^2 = m_{a_1}^2$ ,  $m_{f_1^{(2)}}^2 = m_S^2$  and  $m_{f_1^{(3)}}^2 = m_G^2$ . This is, of course, the limiting case in which the Ellis–Jaffe sum rule [20] is valid and now we have  $\Delta s = 0$ . In particular we can see that there is a direct link between the vanishing of the mixing parameters  $\lambda$  and  $\kappa$  and the vanishing of the strange density moment in the nucleon. At the same time, we find that the singlet sum  $\Delta\Sigma \rightarrow 1$  as  $\lambda, \kappa \rightarrow 0$ . In the case of the pseudoscalar mesons the vanishing of the mixing parameter  $\lambda$  implies that the gluonic anomaly is not present, and there is a degeneracy between the singlet and the octet pseudoscalar mesons. As we have suggested, a similar phenomenon is supposed to occur in the axial vector meson channel. Thus, we can make the gluonic anomaly responsible for a non-vanishing strange quark moment of the nucleon. At present it is not clear whether this phenomenon, which implies a large violation of the Zweig rule in the  $(1^{++})$ –channel, is directly related to the  $U(1)$  problem. At this point it is interesting to consider recent analyses of the singlet axial channel based on the sum rule technique. The authors of reference [21] aim to evaluate  $\Delta\Sigma$  in a way similar to the calculation of the octet axial constant but by taking into account the presence of the anomaly in the singlet axial channel. This analysis leads to the same conclusion as the one above, namely that the singlet axial channel is qualitatively similar to the pseudoscalar channel and differs much from the corresponding situation in the vector channel. In a more recent study [22], the matrix elements arising from the operator product expansion for the deep inelastic scattering amplitude are factorised into composite operator propagators and proper vertex functions. Whereas the vertex is evaluated according to the OZI–rule, the propagator, being RG non–invariant, is computed using QCD spectral sum rules. As a result, this paper also obtains  $\Delta\Sigma$  in agreement with the experimental data and finds its suppression to be a consequence of the anomaly. In a different approach [23],  $\Delta\Sigma$  is calculated in the framework of QCD sum rules using an interpolating nucleon current which explicitly contains the gluonic degrees of freedom. Again,  $\Delta\Sigma$  is obtained in agreement with observation and the conclusions are the same as in reference [22].

Finally we should like to emphasize that the successful description of the axial vector meson situation by our mixing scheme implies that the three observed neutral isosinglet axial vector mesons  $f_1$  are indeed superpositions of  $\bar{u}u/\bar{d}d$ ,  $\bar{s}s$  and gluonic states. As expected, the mixing among the three states is large, i. e., none of the states can be considered to be a pure  $\bar{q}q$  or pure gluonic state. While the results for the mixing parameters estimated by us are subject to a large uncertainty, it is important to note that within our approach the gluonic anomaly manifests itself also in the  $(1^{++})$ -channel. The existence of gluonic states in the  $(1^{++})$ -channel, their mixing with the  $\bar{q}q$ -states and the problem of the nucleon spin are intimately related. This strengthens the idea that the problem of the nucleon spin is intrinsically related to non-perturbative aspects of chiral QCD dynamics.

## 4. CONCLUSIONS

In this paper we have described a mixing pattern within the axial vector meson channel by similar methods as used to describe the mixing in the pseudoscalar meson channel where the gluonic anomaly is operating. We suggest that in the case of the axial vector currents the behavior of the longitudinal part and of the transverse part of the current matrix elements is qualitatively similar, implying a large violation of the Zweig rule also in the axial vector channel. We have shown that a consistent picture emerges provided there are three neutral isosinglet axial vector states, in accordance with the experimental observation. These three states are superpositions of  $\bar{u}u/\bar{d}d, \bar{s}s$  and gluonic states. The mixing among these states is large. It leads to a non-vanishing contribution of the strange density function in the nucleon. The sign of the strange density function is negative because of the fact that the isosinglet state with the smallest mass contributing to the corresponding matrix element is close to an  $SU(3)$  octet, i. e., to the state  $\frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$ . The mixing which is essential in order to arrive at a correct description of the observed spectrum leads at the same time to a non-vanishing polarized strange quark density and to a reduction of the singlet sum  $\Delta\Sigma$ . Since the mixing reflects directly a non-perturbative feature of the gluonic anomaly, we find a direct link between the gluonic anomaly as a non-perturbative feature of the dynamics of the nucleon and the nucleon spin. It remains to be seen whether the large mixing in the axial vector meson channel suggested here can indeed be obtained in taking into account the non-perturbative features of QCD, e. g. in the lattice approach or by considering QCD sum rules. For example, we would expect that the two-point function  $\langle 0|\bar{u}(x)\gamma_\mu\gamma_5u(x)\bar{d}(y)\gamma_\nu\gamma_5d(y)|0\rangle$ , which vanishes in the absence of a gluonic interaction, receives strong contributions not only in the longitudinal part (because of the pseudoscalar gluonic anomaly), but also in the transverse part. These effects should be investigated in more detail using perturbative techniques [24].

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